JOURNAL OF AIRCRAFT
Vol. 29, No. 2, March–April 1992

Preliminary Sizing Methodology for Hypersonic Vehicles

Armand J. Chaput*
General Dynamics Corporation, Fort Worth Division, Fort Worth, Texas 76101

Simplified design and analysis relationships have been developed that are suitable for conceptual-level sizing and synthesis of hypersonic vehicles. Relationships have been developed from point design studies and normalized for generalized vehicle design applications. First- and second-order polynomial curve-fit expressions describe the normalized, aerodynamic, and propulsion performance parameters. Geometry is represented by simple shapes and areas. Mass properties are approximated by unit weights and factors. A simplified atmosphere and equations of motion complete an equation set that can be iterated and solved using elementary programming techniques.

Nomenclature

A_{EF}	= engine frontal area
a	= coefficient defined by Eq. 24 (dimensionless)
b	= wing span, ft
C_{D_L}	= drag due to lift (dimensionless)
$C_{D_{\mathrm{MIN}}}$	= minimum drag coefficient (dimensionless)
$C_{D_{\mathrm{TRIM}}}$	= trim drag coefficient (dimensionless)
C_L	= lift coefficient (dimensionless)
C_r	= wing theoretical root chord, ft
$C_{r_{\mathrm{EXP}}}$	= exposed wing root chord, ft
C_t	= wing theoretical tip chord, ft
D	= vehicle drag, lb
d	= nominal fuselage diameter, ft
E	= total energy parameter, ft^2/s^2
ESF_{HSS}	= engine scale factor (dimensionless)
F_N	= net propulsive thrust, lbf
F_{SP}	= net specific thrust, s
f/a	= stoichiometric fuel-to-air ratio (dimensionless), 0.0293 for H_2 air
ff	= propellant fraction (dimensionless)
g	= standard acceleration due to gravity, ft/s^2
h	= altitude, ft
h_1	= initial incremental altitude, ft
h_2	= final incremental altitude, ft
I_{SP}	= cycle specific impulse, s
$I_{\mathrm{SP}_{EFF}}$	= vehicle specific impulse, s
K	= incremental weight ratio (dimensionless)
k_{HT}	= horizontal tail area ratio (dimensionless)
k_{LG}	= landing gear gross weight fraction (dimensionless)
k_{OF}	= other fluids fraction (dimensionless)
k_{PF}	= propellant packing factor (dimensionless)
k_{RES}	= fuel reserve fraction (dimensionless)
k_{SS1}	= type 1 subsystem-to-empty weight ratio (dimensionless)
k_{SS2}	= type 2 subsystem-to-gross weight ratio (dimensionless)
k_{SWET}	= ratio of actual to approximated fuselage wetted areas
k_{VOL}	= fuselage "photographic" scale factor (dimensionless)
k_{VT}	= vertical tail area ratio (dimensionless)
L	= vehicle lift, lb
L/W	= lift-to-weight ratio (dimensionless)
M	= Mach number (dimensionless)
m	= drag-due-to-lift factor (dimensionless)
N	= number of integration intervals (dimensionless)
q	= freestream dynamic pressure, lb/ft^2
$q_{T/O}$	= takeoff dynamic pressure, lb/ft^2
r_i	= fuselage radius at station x_i, ft
$S_{\mathrm{EXP\ WING}}$	= wing exposed planform area, ft^2
S_{HT}	= horizontal tail planform area, ft^2
S_{REF}	= theoretical wing reference area, ft^2
S_{VT}	= vertical tail planform area, ft^2
S_{WET}	= total vehicle wetted area, ft^2
$S_{\mathrm{WET\ FUS}}$	= fuselage wetted area, ft^2
$S_{\mathrm{WET\ HT}}$	= horizontal tail wetted area, ft^2
$S_{\mathrm{WET\ VT}}$	= vertical tail wetted area, ft^2
$S_{\mathrm{WET\ WING}}$	= wing wetted area, ft^2
$(T/W)_{HSS}$	= high-speed propulsion system thrust-to-weight ratio (dimensionless)
$(T/W)_{LSS}$	= low-speed propulsion system thrust-to-weight ratio (dimensionless)
(T_0/W_0)	= vehicle takeoff thrust-to-weight ratio (dimensionless)
UWF_{FUS}	= fuselage structure unit weight, lb/ft^2
UWF_{HSS}	= high-speed propulsion system unit weight, lb
UWF_{HT}	= horizontal tail structure unit weight, lb/ft^2
UWF_{VT}	= vertical tail structure unit weight, lb/ft^2
UWF_{WING}	= wing structure unit weight, lb/ft^2
V	= volume, ft^3
V_{TANK}	= fuel tank volume, ft^3
v	= freestream velocity, ft/s
v_1	= initial incremental velocity, ft/s
v_2	= final incremental velocity, ft/s
v_{FINAL}	= final ascent velocity, ft/s
v_{INITIAL}	= initial ascent velocity, ft/s
W	= vehicle weight, lb
$\dot{W}_a$	= engine airflow, lb/s
W_E	= empty weight, lb
W_F	= fuel weight, lb
$\dot{W}_f$	= propellant flow rate, lb/s
W_{FINAL}	= final mission weight, lb
W_{FUS}	= fuselage structure weight, lb
W_{HSS}	= high-speed propulsion system weight, lb
W_{HT}	= horizontal tail structure weight, lb
W_{LSS}	= low-speed system propulsion weight, lb
W_{OF}	= weight of other fluids, lb
W_{PAY}	= payload weight, lb
W_{PROP}	= propellant weight, lb
W_{RES}	= reserve propellant weight, lb
$W_{T/O}$	= gross takeoff weight, lb
W_{VT}	= vertical tail structure weight, lb
W_{WING}	= wing structure weight, lb
$W_{L/O}$	= design takeoff wing loading, lb/ft^2

Presented as Paper 87-2954 at the AIAA/ALIS/ASEE Aircraft Design, Systems, and Operations Meeting, St. Louis, MO, Sept. 14–16, 1987; received Oct. 20, 1988; revision received June 29, 1990.

*Chief Engineer, National Aero-Space Plane. Associate Fellow AIAA.

W_1 = initial incremental weight, lb
W_2 = final incremental weight, lb
x_i = fuselage station, ft
α = angle of attack, deg
$\Delta F_{SP_{COOL}}$ = coolant contribution to specific thrust, s
$\Delta\Phi_{COOL}$ = coolant flow fraction of equivalence ratio (dimensionless)
$\Delta v_{1\text{-}2}$ = incremental velocity change, ft/s
δ = freestream atmospheric pressure relative to sea level (dimensionless)
θ = freestream absolute temperature relative to sea level (dimensionless)
λ = wing-taper ratio (dimensionless)
$\bar{\rho}_{PROP}$ = average propellant density, lb/ft^3
σ = freestream atmospheric density relative to sea level
Φ = fuel equivalence ratio (dimensionless)
$\bar{\ }$ = average value over interval

Introduction

AFTER a 20-year hiatus during which only a few stalwarts kept capabilities alive, the country is once again embarked on the development of airbreathing hypersonic vehicles. For those involved in the design and development of these vehicles, coming back up to hypersonic speeds has been a relatively rapid process, due in large part to the impetus provided by major new hypersonic programs such as the National Aero-Space Plane (NASP). However, because of national security considerations and resource limitations, only a few designers will be directly involved in these efforts. Therefore, how do those who are not participants in such programs develop the insights and skills required to execute a design, much less even know where to start?

Fortunately, a number of excellent texts are available in specific hypersonic disciplines. Of these, the most useful for design purposes are in the field of aerodynamics. Excellent propulsion- and structure-related texts are also available, but they are usually oriented toward functional specialists and are sometimes difficult to reduce to vehicle design practice. Almost nonexistent are texts on hypersonic vehicle design. However, many excellent design studies from the 1960s and early 1970s provide data on specific vehicle concepts. Although dated, these studies provide excellent baselines from which contemporary applications can be developed. In addition, the design data presented can be normalized and used to synthesize a wide range of state-of-the-art derivatives.

Analytical Approach

Specific baseline vehicle design data are reduced to a normalized form using parametrics that model primary design drivers (thrust, drag, and weight) as functions of size, shape, and trajectory. Other drivers such as volume, cooling requirements, and maneuver loads can also be included as required. The combined effects of the individual parametrics are linked by simplified atmospheric and vehicle performance models and evaluated numerically using elementary programming techniques (an iterative, multivariable version of the sizing/synthesis procedure described in this paper was executed on a programmable 16K pocket computer).

Application of the parameters to vehicles that may vary significantly from the baseline are accommodated by changes in geometric input or by adjusting design parameters (i.e., modifying structural unit weights to capture the impact of a structural design change). Other changes, such as technology advancements, can be incorporated as multipliers (i.e., a percentage reduction in structural unit weight to represent advanced material substitution) and appropriate sizing and performance impacts determined. More complicated changes such as fuel or engine cycle changes require development of new parameters that can be added to an appropriately modularized computational procedure.

Baseline Approach

Minimum requirements for a baseline vehicle are that the vehicle should be 1) representative of the configuration type under consideration and 2) described by a relatively complete technical data package. A conventional wing-body-tail design (Fig. 1) was selected as the example for this paper because of

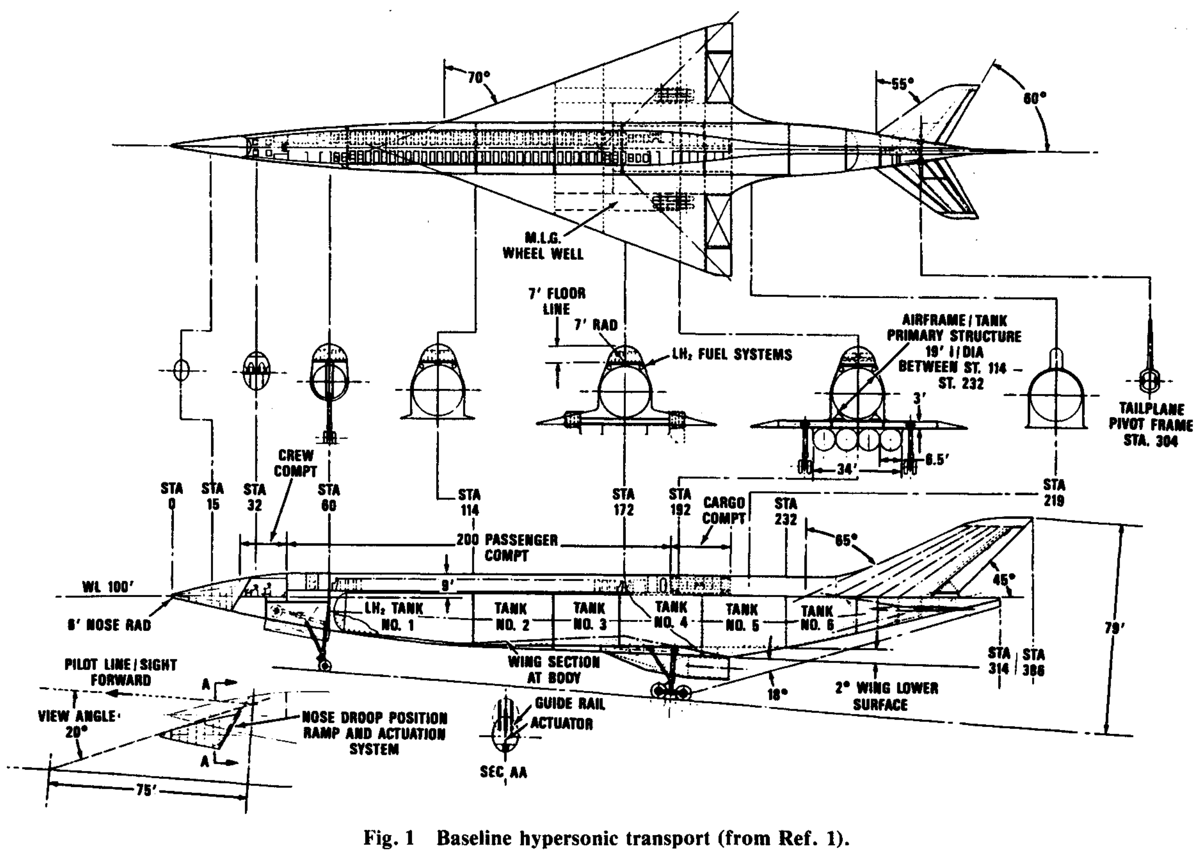

Fig. 1 Baseline hypersonic transport (from Ref. 1).

the interest in this configuration type and completeness of its technical documentation. The candidate vehicle was, however, only one of a number of hypersonic transport (HST) configurations evaluated in Ref. 1 that could have been selected.

The baseline HST is designed to carry 200 passengers and is powered by four hydrogen-fueled Pratt & Whitney turboramjet study engines mounted externally to the basic vehicle structure. The engines are sized for uninstalled sea-level static thrust of 86,800 lbf each.

A geometric description of the baseline is contained in Table 1. Weights are contained in Table 2. Figure 2 shows the design mission profile and the constraints that determined the limits of the operational envelope. Because of the prevailing interest in accelerator-type vehicles, the synthesis/sizing procedure was coded and applied only up to the end of the ascent portion of the design mission profile ($M = 6.0$, $h = 103{,}000$ ft). Cruise, descent, loiter, and landing requirements were characterized by a single, postascent fuel reserve.

Table 1 Baseline HST geometry

Wing	
Reference area S_{REF}	6630 ft^2
Span b	98 ft
Root chord C_r	134.7 ft
Tip chord C_t	0 ft
Aspect ratio AR	1.45
Exposed area $S_{EXP\ WING}$	3880 ft^2
Taper ratio λ	0
Horizontal tail	
Area S_{HT}	1207 ft^2
Vertical tail	
Area S_{VT}	1079 ft^2
Fuselage	
Length l	314.5 ft
Nominal diameter d	20.9 ft
Total volume V_{TOTAL}	83,418 ft^3
Tank volume V_{TANK}	46,416 ft^3
Wetted area S_{WET}	19,863 ft^2
Propulsion	
Engine frontal area A_F	113 ft^2

Table 2 Baseline HST weights, lb

Structure		162,597
Wing	38,228	
Horizontal tail	7,914	
Vertical tail	7,451	
Fuselage (including tanks)	109,004	
Propulsion		62,053
Engine and acc.	41,408	
Air induction	16,967	
Nacelles and structure	3,678	
Landing gear		16,385
Subsystems		43,192
Fuel	1,389	
Pressurization and lubrication	4,049	
Secondary power and electric Distribution	7,966	
Controls	5,230	
Subtotal (type 1)	18,634	
Instruments and avioncs	3,224	
Environmental control	7,606	
Passenger and cargo Provisions	13,428	
Crew provisions	300	
Subtotal (type 2)	24,558	
Empty weight		284,227
Payload and crew		49,275
Fluids		203,538
Fuel	196,646	
Residuals and other fluids	1,638	
Losses	5,254	
Gross takeoff weight		537,040

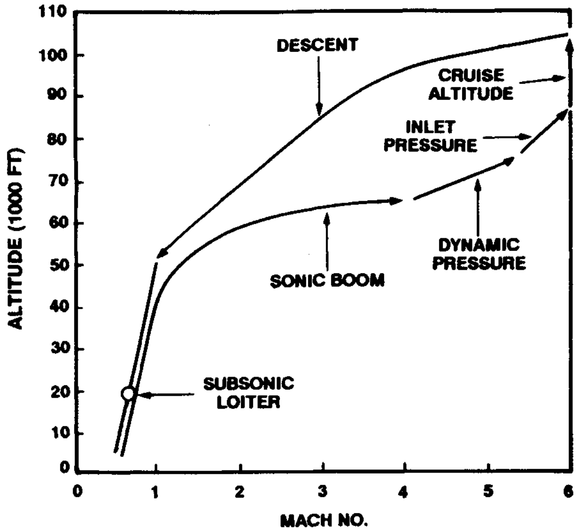

Fig. 2 HST trajectory design constraints.

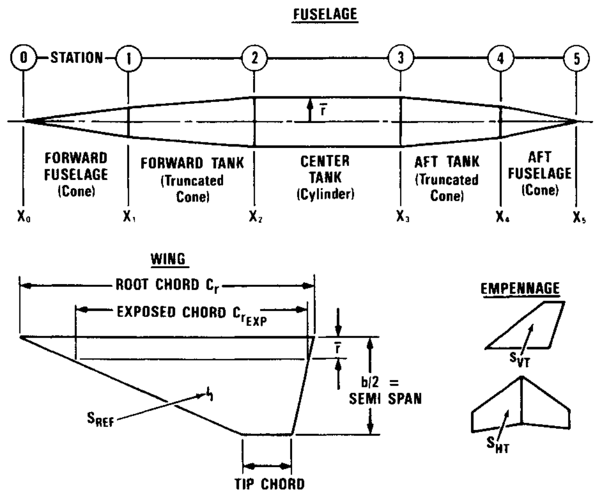

Fig. 3 Vehicle geometry approximations.

Vehicle Geometry Relationships

Simple geometric shapes are used to represent major components of the baseline vehicle. To maintain geometric similarity with the baseline, fuselage geometry changes are limited to "photographic" scaling. The wing/empennage and fuselage, however, are scaled independently to decouple planform and volume effects. Simple wetted and/or planform area ratios are used to correct baseline parameters for scale effects. Major deviations from the baseline configuration require careful assessment of input parameters and may result in rederivation in some areas.

Fuselage Definition

For preliminary sizing purposes, reasonable accuracy is achieved by analytically approximating the vehicle fuselage as a series of cones and cylinders (Fig. 3). The fuselage is assumed to contain three fuel compartments that are described by forward and aft truncated cone sections and one connecting cylindrical center section. Cone shapes form the forward and aft fuselage closures. Nonfuel fuselage volume allocations are modeled directly or approximated by appropriately reduced fuel packing factors. In addition, limits are placed on appropriate dimensions (i.e., fixing the size of the forward cone to prevent the crew compartment from scaling below reasonable limits).

Handbook procedures provide the following generalized equations for the approximated wetted areas and volumes:

$$S_{\text{WET FUS}} = \Pi \sum_{i=1}^{5} (r_{i-1} + r_i)\sqrt{(x_{i-1} - x_i)^2 + (r_{i-1} - r_i)^2}$$
$$V_{\text{TANK}} = \frac{\Pi}{3}\sum_{i=2}^{4} (x_i - x_{i-1})(r_{i-1}^2 + r_i r_{i-1} + r_i^2) \tag{1}$$

Packing factors provide corrections for tank shape variations, voids, and nonoptimums. Similarly, wetted area multipliers are applied to correct for noncircular shapes and protuberances (i.e., passenger compartments and engine cowls).

Baseline vehicle dimensions were scaled from Fig. 1 and used to calculate approximate $S_{\text{WET FUS}}$ and V_{TANK}. Ratios of these approximate values to those contained in Ref. 1 were used to calculate wetted area (k_{SWET}) and packing factor (k_{PF}) corrections of 1.35 and 0.78, respectively. These factors not only provide corrections for simplified geometric approximations but also can be used as design parameters to evaluate such effects as vehicle density and configuration-dependent wetted area-to-volume variations. Other volume requirements and sensitivities such as those associated with specific propulsion or subsystem concepts can also be incorporated as sizing parameters.

Wing Definition

A simple, trapezoidal shape (Fig. 3) is used to approximate wing planform geometry in terms of wing span b, root chord C_r, and taper ratio λ:

$$S_{\text{REF}} = \frac{bC_r}{2}(1 + \lambda) \tag{2}$$

Exposed wing area is approximated by

$$S_{\text{EXP WING}} = \frac{bC_r}{2}\left[(1 + \lambda) - 2\left(\frac{d}{b}\right) + (1 - \lambda)\left(\frac{d}{b}\right)^2\right] \tag{3}$$

For the baseline vehicle, the equation yields approximate values for S_{REF} and $S_{\text{EXP WING}}$ of 6600 and 4035 ft^2, respectively.

Wing wetted area is approximated at twice the exposed planform area. If desired, wing volume available for fuel can be included for a nominal airfoil shape and thickness ratio. However, for hydrogen-powered vehicles, such as the baseline, thin wings do not provide adequate volume for fuel containment.

Empennage Definition

Horizontal and vertical empennage planform areas are expressed as percentages of the wing reference area. Wetted areas are approximated by doubling the planform areas. The generalized equation is

$$S_{HT} = k_{HT} S_{\text{REF}}$$
$$S_{VT} = k_{VT} S_{\text{REF}} \tag{4}$$

For the baseline, k_{HT} and k_{VT} are determined to be 0.18 and 0.16, respectively.

Volume Required

To maintain consistency with baseline characteristics, fuselage volume required is determined by "photographic" scaling, i.e., all fuselage dimensions are scaled equally. The scaling parameter can be approximated by the cube root of the ratio of required-to-available fuel volume as

$$k_{\text{VOL}} = \sqrt[3]{\frac{V_{\text{TANK(REQ'D)}}}{V_{\text{TANK(AVAIL)}}}} \tag{5}$$

Weight Relationships

Vehicle mass properties are broken down into categories consisting of structural, propulsion, landing gear, subsystems, payload (including crew), and fuel weights.

Structural Weights

Fuselage, wing, and empennage weights are assumed to be primarily wetted-area dependent. Unit weight factors (UWFs) are used to estimate these area-dependent weights as

$$W_{\text{FUS}} = UWF_{\text{FUS}} S_{\text{WET FUS}}$$
$$W_{\text{WING}} = UWF_{\text{WING}} S_{\text{WET WING}}$$
$$W_{HT} = UWF_{HT} S_{\text{WET HT}} \tag{6}$$
$$W_{VT} = UWF_{VT} S_{\text{WET VT}}$$

The UWFs were calculated for the baseline from Tables 1 and 2 as

$$UWF_{\text{FUS}} = \frac{109{,}004 \text{ lb}}{19{,}863 \text{ ft}^2} = 5.5 \text{ lb/ft}^2$$
$$UWF_{\text{WING}} = \frac{38{,}228 \text{ lb}}{2 \times 4035 \text{ ft}^2} = 4.7 \text{ lb/ft}^2$$
$$UWF_{HT} = \frac{7914 \text{ lb}}{2 \times 1207 \text{ ft}^2} = 3.3 \text{ lb/ft}^2$$
$$UWF_{VT} = \frac{7451 \text{ lb}}{2 \times 1079 \text{ ft}^2} = 3.5 \text{ lb/ft}^2$$

Note that UWF_{WING} is derived using the calculated value from Eq. (3) and that wing and empennage area is approximated as twice that of the appropriate planform.

Propulsion Weights

Two propulsion system types are normally associated with hypersonic vehicles: low speed (for takeoff and acceleration to ramjet speeds) and high speed (for acceleration from supersonic to hypersonic speeds and to sustain cruise). Because of the different requirements on the two systems, it is useful to size and weigh each separately. The following sizing equations are used:

$$W_{LSS} = (T_0/W_0)\frac{W_{T/O}}{(T/W)_{LSS}}$$
$$W_{HSS} = ESF_{HSS}(UWF_{HSS}) \tag{7}$$

The values derived or given for the baseline vehicle were $T_0/W_0 = 0.65$, $(T/W)_{LSS} = 8.7$, $ESF_{HSS} = 5.39$, and $UWF_{HSS} = 5127$ lb.

In developing the above values, it was necessary to allocate the total propulsion system weight of 62,053 lb between the low-speed or high-speed systems. For simplicity, nominal ratios were used and resulted in allocations of $W_{LSS} = 37{,}137$ lb and $W_{HSS} = 24{,}916$ lb.

Subsystem Weights

Simple ratios are used to define subsystem weights. Some systems, however, are assumed to be driven primarily by empty weight, whereas others will be driven primarily by gross weight. The control system is an example of a subsystem that is assumed to be sized primarily by gross weight, whereas the environmental control system is assumed to be driven mostly by empty weight. The subsystem factors arbitrarily defined as type 1 and type 2, respectively, for the baseline vehicle are $k_{SS1} = 0.065 \times W_E$ and $k_{SS2} = 0.046 \times W_{T/O}$. Table 2 shows subtotals that depict the assumed breakdown by type. A landing gear factor defined by the ratio of the sum of the nose and main gear weights relative to gross weight resulted in $k_{LG} = 0.03$.

Fluid Weights

Fluids include primary propellants, reserves, residuals, and other fluids. Propellants, reserves, and residuals are defined in terms of tank volume available, fuel density, and packing factor. Other fluids are defined as fixed percentages. The following equations define the various fluid types:

$$W_{\mathrm{PROP}} = \bar{\rho}_{\mathrm{PROP}} k_{PF} V_{\mathrm{TANK}}$$

$$W_{\mathrm{RES}} = k_{\mathrm{RES}} W_{\mathrm{PROP}}$$

$$W_{OF} = k_{OF} W_{\mathrm{PROP}} \tag{8}$$

The following numerical values were derived or given for the baseline: $\bar{\rho}_{\mathrm{PROP}} = 4.4$ lb/ft^3, $k_{PF} = 0.78$, $k_{\mathrm{RES}} = 0.66$, $k_{OF} = 0.04$.

The large reserve propellant fraction k_{RES} results from simulating only the takeoff and ascent segments of flight as described previously. The propellant packing factor k_{PF} has already been calculated (see Fuselage Definition). Note also that if multiple propellants are used (i.e., oxidizers), $\bar{\rho}_{\mathrm{PROP}}$ will become an iteration variable due to density differences that drive volume requirements.

Fixed Weights

This category includes crew, payload, and provisions that do not scale with vehicle size. The following values from Ref. 1 were given for crew and provisions at 1275 lb and cargo and passengers at 48,000 lb. For simplicity, the values are summed to yield a single "payload" weight of $W_{\mathrm{PAY}} = 49{,}275$ lb.

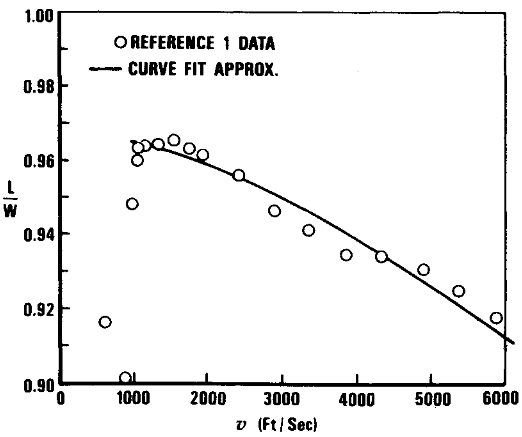

Fig. 4 Lift-to-weight approximations.

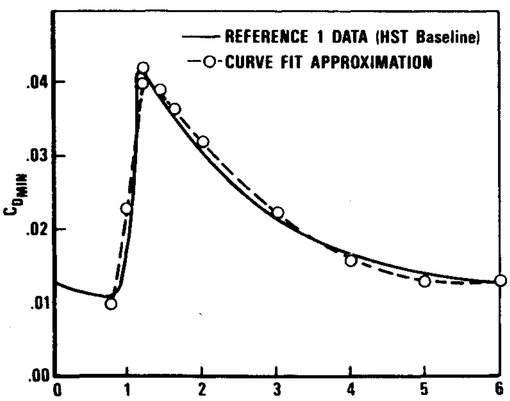

Fig. 5 Baseline minimum drag approximation.

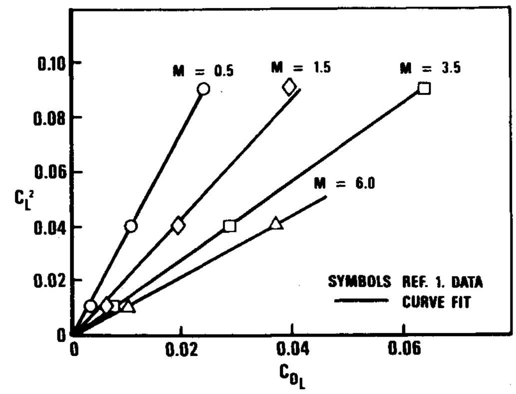

Fig. 6 Baseline drag-due-to-lift approximation.

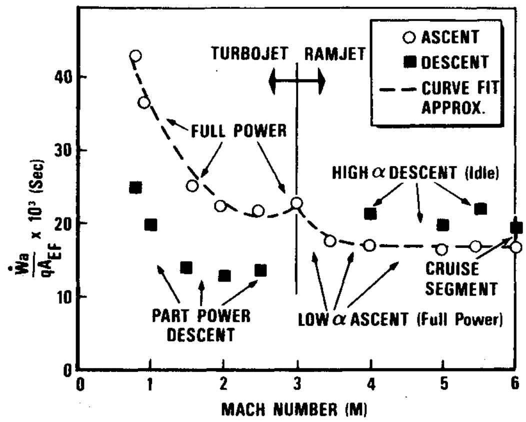

Fig. 7 Baseline vehicle airflow parametric.

Aerodynamic Relationships

Derivation of generalized hypersonic vehicle aerodynamic relationships is beyond the scope of this paper. Instead, simplified relationships based only on weight, wing reference area, total wetted area, and dynamic pressure are used.

Lift

Forces perpendicular to the flight path vector generally considered significant are wing/body lift and the normal components of thrust, weight, and centrifugal acceleration. Calculation of these forces requires an iterative solution that is computationally manageable but is also beyond the scope of this paper. However, for relatively small angles of attack, reasonable accuracy can be achieved by calculating a nominal correction factor in the form of a velocity-dependent multiplier derived from analysis of a baseline vehicle. The multiplier is derived by performing a curve fit of lift-to-weight ratio to velocity (Fig. 4). This approach yields relatively good accuracy over much of the high-speed envelope. Significant errors, however, will occur at low speeds and on high-acceleration trajectories.

Drag

Vehicle drag is assumed to consist of two components expressed in coefficient form as

$$D = qS_{\mathrm{REF}}[C_{D_{\mathrm{MIN}}}(M) + C_{D_L}(M)C_L^2] \tag{9}$$

Other drag components, including trim, can be modeled as scalar multipliers (i.e., $C_{D_{\mathrm{TRIM}}} \approx 5\%$).

The aerodynamic coefficients for the baseline vehicle are shown in Figs. 5 and 6. Simple curve-fitting techniques were used to generate numerical approximations. The $C_{D_{\mathrm{MIN}}}$ is approximated by first- and second-order functions over discrete Mach number segments to simplify the nonlinear equations. A two-variable, least-squares fit to the data (Fig. 7) provides a simplified relationship for drag due to lift. For scaling purposes, $C_{D_{MIN}}$ is assumed to vary linearly with total wetted area. The C_{D_L} is assumed to remain constant, since basic wing parameters (i.e., aspect ratio and sweep) are unchanged. Engine-related drag is included in the net thrust definition used by the propulsion relationships in the following section.

Propulsion Relationships

Different propulsion models are required for low-speed (takeoff and acceleration to supersonic speeds) and high-speed (ramjet and/or scramjet) systems. The former typically are adaptations of existing turbomachines and their performance can be approximated accordingly. Ramjets and scramjets, however, are fundamentally different cycles and other methodology must be applied. Fortunately, this methodology is relatively straightforward (at conceptual levels) since the performance of these engines tends to depend primarily on airflow. Turbomachinery performance is also a strong function of airflow but is more dependent on throttle setting and is less sensitive to vehicle attitudes.

Airflow Characteristics

First-order approximations of both low- and high-speed airflow are developed by correlating airflow ($\dot{W}_a$) per unit engine frontal area times q as shown in Fig. 7. These data from the baseline HST cover a wide range of dynamic pressures (from a few hundred psf to over 2000) but still normalize in a relatively well-behaved fashion over the Mach range. This is particularly true for ascent where maximum power throttle settings are maintained and vehicle attitude changes are small. The baseline engine scale factor (ESF_{HSS}) is defined as unity (1.00) for a frontal area (A_{EF}) of 113.1 ft^2.

During descent, considerable throttling occurs for the baseline mission and substantial angle-of-attack excursions α are experienced. Although the baseline data in Ref. 1 are not sufficiently comprehensive to rigorously verify the relationship, they do show that the influence of α on $\dot{W}_a$ is independent of throttle setting. The data also demonstrate the relative insensitivity of turbomachinery performance to α and that ascent-descent variations are primarily due to differences in throttle setting. Note also that the airflow parameter ($\dot{W}_a/qA_{EF}$) clearly shows the effect of ramjet transition both on ascent and descent.

Since this paper addresses only the ascent portion of flight where angle-of-attack variations are small and power settings are constant, α and throttling relationships are not presented. They were derived, however, and found to be computationally manageable. Nonetheless, for the ascent phase, reasonable accuracy is achieved by ignoring these effects and correlating ($\dot{W}_a/qA_{EF}$) with M only (Fig. 7). For simplicity, the correlation is defined by separate polynomial approximations over discrete Mach ranges.

Fuel Flow Characteristics

Fuel flow ($\dot{W}_f$) and airflow ($\dot{W}_a$) are related by

$$\dot{W}_f = (f/a)\Phi\dot{W}_a \tag{10}$$

Stoichiometric combustion is defined as an equivalence ratio $\Phi = 1$. Throttling and/or cooling requirements, however, can result in lean ($\Phi < 1$) or rich ($\Phi > 1$) mixture operations. The baseline HST speed regime in Ref. 1 is relatively low, and no excess cooling flows are required, except for cruise. For vehicles that fly at higher speeds, a Φ schedule as a function of M and q will be required to model ascent performance accurately.

Thrust Characteristics

Thrust available is approximated by a specific thrust (F_{SP}) and Φ parametric where F_{SP} by definition is given as

$$F_{SP} = (F_N/\dot{W}_a) \tag{11}$$

The parametric has been found to be relatively insensitive to q variations and to vary primarily with M. Furthermore, the effects are approximately linear over the ranges $\Phi < 1$ and $\Phi > 1$.

The characteristic Mach number variation is shown by the ascent data in Fig. 8. The small differences between ascent and descent show the validity of the linear parametric relationship for $\Phi < 1$. For $\Phi > 1$, however, F_{SP} varies according to a different linear relationship.

Equivalence ratios are typically greater than unity in speed regimes where more cooling flow is required than can be accommodated by stoichiometric combustion. Because of airflow limits, the excess fuel does not combust, but it does have a thrust contribution. Part of this is attributable to the momentum of the excess fuel. At an injection temperature of 2000°R (near engine material temperature limits), each additional equivalence ratio of coolant contributes about 13 s of specific thrust, or

$$\Delta F_{SP_{COOL}} = 13.0(\Delta\Phi_{COOL}) \tag{12}$$

where

$$\Delta\Phi_{COOL} = \Phi - 1 \tag{13}$$

For purposes of comparison, this relationship translates into a specific impulse for the coolant of about 445 s.

In Fig. 8, the performance of the turbojet low-speed system is also represented by an F_{SP} function. Its throttled performance, however, is more complex than the ramjet and does not reduce to a simple Φ relationship. If a rigorous performance simulation is required for throttled turbomachinery, more complex modeling techniques will be needed.

Propulsion Scaling

Engine weights (and volumes if available) can also be assumed to vary linearly with engine size (Fig. 9). However, the

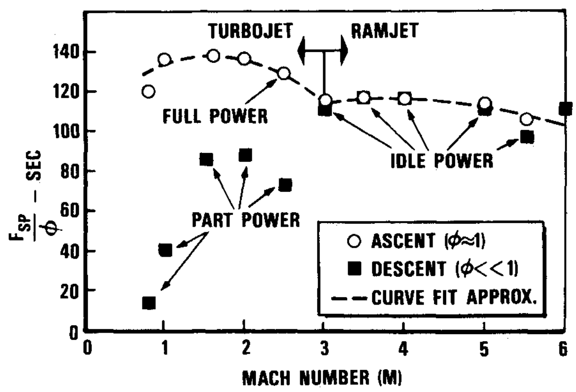

Fig. 8 Baseline vehicle specific thrust (F_{SP}) characteristics.

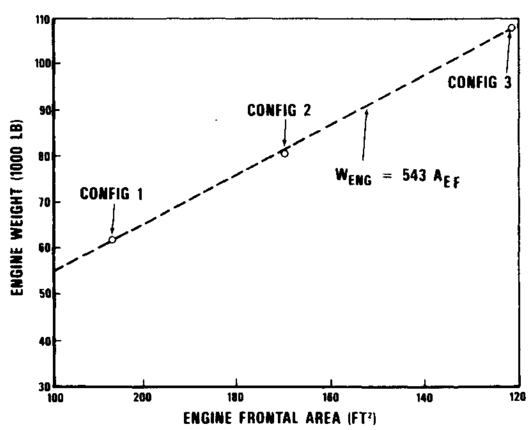

Fig. 9 Engine weight scaling.

baseline scaling relationship is highly dependent on engine design and materials and does not apply universally. It does, however, provide a benchmark value against which technology trades can be performed.

Performance Modeling

Performance is modeled separately for each of the following baseline HST flight regimes described in Ref. 1: 1) takeoff and initial climb, 2) ascent, 3) cruise, 4) descent, 5) loiter, and 6) landing.

Simplified relationships can be developed to model all six regimes but, as described previously, only takeoff and ascent are quantified in this paper. Modeling techniques for the other regimes will be similar with the only unique modeling requirement being for throttled turbojet performance.

Takeoff and Initial Climb

This mission segment typically sizes wing area and the low-speed system. A number of parametrics are available that correlate performance in this regime. Reference 3 provides an example based on handbook data from over 50 military and civil aircraft.

Fuel consumption for takeoff and initial climb is estimated as a fixed percentage of vehicle gross weight. Although a simplified assumption, three fundamentally different size and weight configurations from Ref. 1 were evaluated, and all required fuel quantities equivalent to 1.7% of gross weight to reach $M = 0.8$ (a nominal ascent trajectory starting point).

Ascent

Ascent performance can be modeled with good accuracy using a simplified form of the basic accelerator equation:

$$\frac{\Delta v_{1-2}}{g} = \frac{\bar{F}_N - \bar{D}}{\bar{\dot{W}}_F} \ln\left(\frac{W_1}{W_2}\right) \tag{14}$$

The term $(F_N - D)/\dot{W}_F$ is defined as effective special impulse (ISP_{EFF}). A less familiar form that is consistent with the propulsion parametrics presented herein results from combining Eqs. (10), (11), and (14) and yields

$$\Delta v_{1-2} = \frac{g}{(f/a)}\frac{1}{\bar{\Phi}}\left[\bar{F}_{SP} - \frac{\bar{D}}{\bar{\dot{W}}_a}\right]\ln\left(\frac{W_1}{W_2}\right) \tag{15}$$

Since F_{SP}, Φ, D, and $\dot{W}_a$ are functions of trajectory variables q and M, a series of incremental calculations can be performed to approximate the complete trajectory and yields the following equation for final trajectory velocity:

$$v_{FINAL} \approx v_{INITIAL} + \frac{g \ln[1/(1-ff)]}{N(f/a)}\sum_{i=1}^{N}\frac{1}{\Phi_i}\left[\bar{F}_{SP_i} - \frac{\bar{D}_i}{\bar{\dot{W}}_{a_i}}\right] \tag{16}$$

where

$$ff = (W_{T/O} - W_{FINAL})/W_{T/O} \tag{17}$$

The approximate equality symbol is used in Eq. (16). For simplicity, it uses the initial value of the subscripted terms for the summation in lieu of the more exact average value over the interval. Also, the simplified equation excludes altitude (i.e., potential energy) effects. This omission can be the source of some error, and a correction factor is presented in the following section.

Regardless of the approximate nature of the equation, its form (and subsequent computation) is simplified considerably by defining summation intervals in terms of a fixed gross weight variation K where

$$K = (W_i/W_{i+1}) \qquad i = 1,N \tag{18}$$

Altitude Correction

The simplified acceleration equation neglects the change in potential energy associated with altitude variations. When corrections are included, the equation for a single summation interval can be approximated by the following specific energy formulation:

$$v_2^2 + 2gh_2 \approx \left[g\left(\frac{F_{N_1} - D_1}{\dot{W}_{f_1}}\right)\ln(K) + v_1\right]^2 + 2gh_1 \tag{19}$$

Note that when the bracketed term is large compared to $2g\Delta h$, the error of omission is negligible. Early in the flight where climb rates are high, the error introduced can be substantial. The problem with this effect, however, is not the form of the equation but rather that it requires definition of an altitude-velocity profile and iteration to solve. Also, a velocity-altitude profile is cumbersome for design definition purposes. A more meaningful definition is one based on parameters, such as q and M, that have physical significance for sizing vehicle structure and systems. This can be done using alternate formulations that correlate q and M with velocity and altitude.

By definition, air density varies with atmospheric pressure and temperature according to the relationship

$$\sigma = (\delta/\theta) \tag{20}$$

In standard English units q and M, by definition, are given as

$$q = 1481\delta M^2 \tag{21a}$$

$$M = v/1116.4\theta \tag{21b}$$

The equations can be rearranged to provide expressions that relate standard trajectory variables q and M with atmospheric properties as follows:

$$(\delta/\theta) = 841.56(q/v^2) \tag{22a}$$

$$\theta = \frac{1}{M^2}\left(\frac{v}{1116.4}\right)^2 \tag{22b}$$

Simplified Atmosphere

Typically, a standard atmosphere is described by discontinuous δ and θ functions expressed algebraically for discrete altitude ranges. One can, however, derive a single approximate relationship that covers the range from sea level to 200,000 feet. The expression can be derived using curve-fitting techniques and is given without proof as follows for an approximate form of a 1962 standard atmosphere:

$$h(1000\text{ ft}) = a - \theta^{(2.112 + 0.05183a - 0.004043a^2)} \tag{23}$$

where

$$a = 9.4113 - 21.824\ln(\delta/\theta) \tag{24}$$

Trajectory Modeling

The previously derived thrust, drag, and atmosphere approximations are used to predict velocity and altitude changes over constant incremental ratios of weight change. An approximate $q = f(M)$ schedule for the baseline vehicle trajectory is shown in Fig. 10.

The simulation starts at initial point 1, where all vehicle (i.e., F_N, D, L/W, $\dot{W}_a$, ϕ, etc.) and atmospheric variables are known. The energy state associated with the next velocity and altitude condition [left side of Eq. (19)] is calculated as follows:

$$v_2^2 + 2gh_2 = E_2 \tag{25}$$

Next, an iterative procedure is used to calculate the altitude and velocity condition that meets the input trajectory defini-

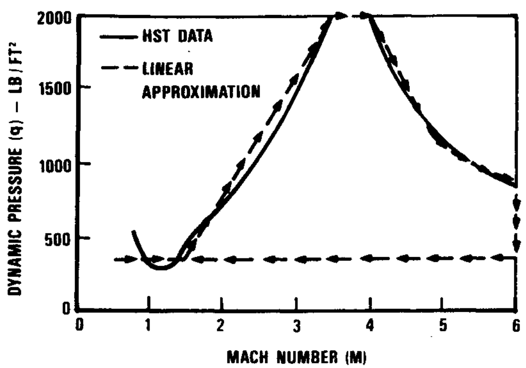

Fig. 10 Baseline vehicle trajectory approximation.

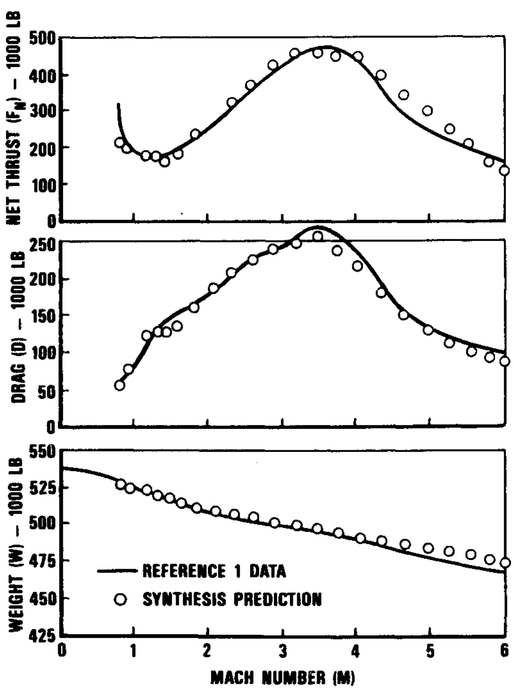

Fig. 11 Comparison of trajectory prediction results.

tion $q = f(M)$ (additional information on the mechanics of this calculation is provided in Ref. 3). Typically, convergence occurs quickly resulting in definition of a new altitude and velocity that, in turn, enables specific thrust, airflow, drag, and equivalence ratio to be recomputed from the previously developed parametrics. A third energy state can then be predicted and another altitude and velocity iterated. This process continues until a prescribed final velocity is achieved. At this point, the takeoff and ascent segments have been completely modeled. Figure 11 shows the results obtained for the baseline compared to those presented in Ref. 1. The differences, though acceptable for preliminary sizing purposes, are due primarily to the simplified Fig. 10 approximation of the relatively complex $q = f(M)$ schedule defined by Fig. 2. If the baseline had flown a less complicated trajectory, a much better correlation would have resulted. This has been verified by comparisons with exact three-degree-of-freedom solutions for vehicles flying constant q trajectories.

Vehicle Convergence and Resizing

Upon completion of the ascent phase (or mission completion in the case of a full simulation), the volume of fuel expended plus all required reserves are compared to the volume of fuel available. A new vehicle scale factor required is calculated using Eq. (5). If the volume change required is within predetermined accuracy limits, the simulation is considered converged. If not, the vehicle is "photographically" resized.

High-speed engine rescaling can be performed by using a number of criteria to include a specified design thrust-to-drag ratio, a constant vehicle frontal-to-engine capture-area ratio, or others. The low-speed system is scaled separately based on design takeoff or landing thrust-to-weight ratio, acceleration rate, or other criteria. In either event, an iterative sizing/weight procedure is required, and the synthesis loop is repeated until all parameters are within prescribed convergence criteria.

Conclusions

Simple linear and second-order parametrics can be used to model the important thrust, drag, weight, and fuel flow characteristics of a typical hypersonic vehicle. These equations are sufficiently tractable that they can be coded, including iteration and resizing loops, to provide the individual engineer with a pocket version of a preliminary synthesis code. Furthermore, by appropriate modifications to the synthesis input and/or algorithms, a range of configuration and technology alternatives can be evaluated.

References

[1]Jarlett, F. A., "Performance Potential of Hydrogen Fueled, Airbreathing Cruise Aircraft," Vol. 2—Phase I Studies, General Dynamics Convair Division, GD/C-DCB66-004/2, San Diego, CA, May 1966.

[2]Pratt & Whitney Aircraft, *Aeronautical Vest-Pocket Handbook,* 16th ed., June 1977.

[3]Chaput, A. J., "Preliminary Sizing Methodology for Hypersonic Vehicles," AIAA Paper 87-2954, Sept. 1987.